PHYS 301 – Assignment #2

Due Wednesday, Oct. 16 at 14:00

1(a) Which of the following is *not* and valid electrostatic field?

$$\mathbf{E} = k\left[xy\,\hat{x} + 2yz\,\hat{y} + 3xz\,\hat{z}\right],$$
$$\mathbf{E} = k\left[y^2\,\hat{x} + \left(2xy + z^2\right)\,\hat{y} + 2yz\,\hat{z}\right]$$

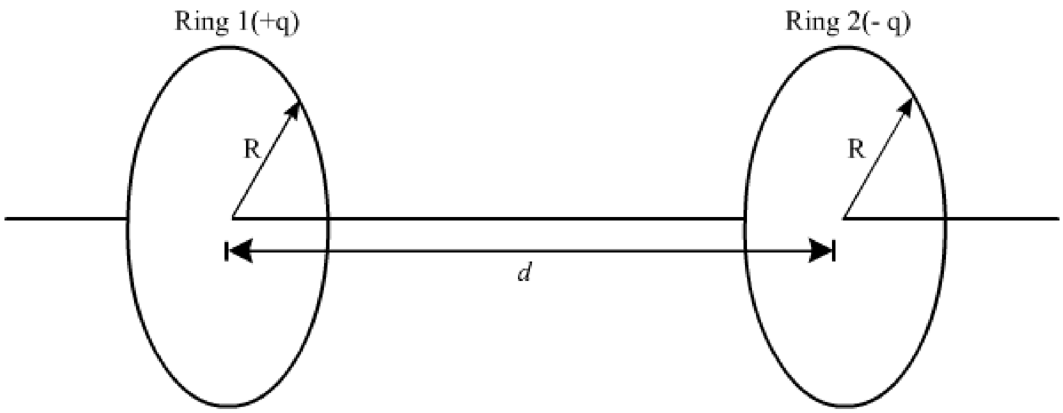
The coefficient k is a nonzero constant.

(b) For the valid electrostatic field, find an expression for the corresponding electric potential $V(x, y, z)$.

2(a) Consider a uniformly-charged ring or radius R and total charge Q. Find the electric potential at a point P that lies on an axis that passes perpendicularly through the centre of the ring. Assume that P is a height z above the plane of the ring.

(b) Find the electric field at P due to the charged ring.

(c) Next, consider a pair of identical, but oppositely charged rings. The rings share a common axis that (say the z-axis) and are parallel to one another. Assume that the rings are separated by a distance d. See the figure below.



Show that the net electric field E at a point on the axis that is a distance $z = xd$ from "Ring 1" and a distance $z = (1-x)d$ from "Ring 2" can be expressed as:

$$E = E_0 \left\{ \frac{x\dfrac{d}{R}}{\left[1+\left(x\dfrac{d}{R}\right)^2\right]^{3/2}} + \frac{(1-x)\dfrac{d}{R}}{\left[1+\left((1-x)\dfrac{d}{R}\right)^2\right]^{3/2}} \right\},$$

where $E_0 = \dfrac{Q}{4\pi\varepsilon_0 R^2}$ and $0 < x < 1$.

(d) For $d = 2.5R$, plot E/E_0 versus x on the interval $0 < x < 1$. Notice that, for points midway between the rings, E is nearly constant along the z-axis.

3(a) Consider a uniformly-charged solid cylinder of radius R and length L. Take the origin of your coordinate system to be at the centre of the cylinder such that a length $L/2$ is above the xy-plane and the other half is below the xy-plane. Starting from:

$$\mathbf{E} = \frac{1}{4\pi\varepsilon_0} \int \frac{\rho\left(\mathbf{r}'\right)}{\mathscr{r}^2} \hat{\mathscr{r}} \, \mathrm{d}\tau',$$

show that the electric field at a point P on the z-axis with $z > L/2$ can be expressed as:

$$\mathbf{E} = \frac{\rho}{2\varepsilon_0} \left[\sqrt{\left(z - L/2\right)^2 + R^2} - \sqrt{\left(z + L/2\right)^2 + R^2} + L\right] \hat{z}.$$

(b) If you're looking for a challenge, show that the expression for $\mathbf{E}$ in (a) reduces to that of a point charge in the limit $z \gg L, R$. This part of the problem won't be graded.